



# Statistical tests



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# Objectives

- Test the normality of quantitative data
  - Test the difference of the means using parametrical tests
- 

# Statistical tests

Test the normality of the data





# Statistical tests

- All the time when we analyze the population based on samples, we must apply statistical tests in order to sustain our findings
- Define hypotheses, apply the test, interpret p
  - To choose the appropriate test for a practical condition
    - The goal of the test should be appropriate
    - The specific conditions of the test must be met



# Research

1. Formulate the hypotheses (fundamental research, documentation, etc.)

2. Experiment (sample)

3. Analyzing data

- The statistical test will be used to
  - Test the internal quality of the sample
  - Verify if the hypotheses is confirmed in the sample

Measurement type	Continuous, parametric	Nominal/ordinal/ nonparametric	Dichotomous (two possible outcomes)	Survival (time to event) ( <i>not clear</i> )
Describe one group	Mean, SD	Median, percentiles	Proportion	Kaplan–Meier survival curve, median survival
Compare one group to a hypothetical value	One sample <i>t</i> -test	Wilcoxon test	Chi-squared or binominal test	
Compare two unpaired groups	Unpaired <i>t</i> -test	Mann–Whitney	Fisher's or Chi-squared	Log rank or Mantel–Haenszel
Compare two paired groups	Paired <i>t</i> -test	Wilcoxon	McNamara's	Conditional proportional hazards regression
Compare three or more unmatched groups	One way ANOVA	Kruskal–Wallis	Chi-squared	Cox proportional hazards regression
Compare three or more matched groups	Repeated-measured ANOVA	Friedman	Cochrane Q	Conditional proportional hazards regression
Quantify relationship between two variables	Pearson correlation	Spearman correlation	Contingency coefficients	
Predict value from another variable	Linear (or nonlinear) regression	Nonparametric regression	Simple logistic regression	Cox proportional hazards regression
Predict values from several measured or binominal variables	Multiple linear (or nonlinear) regression		Multiple logistic regression	Cox proportional hazards regression

*Source:* This table is derived from Mikulski, H. (1995). *Intuitive Statistics*. Oxford Press. New York.

# Testing hypothesis

- Define hypothesis
- Choose the statistical test
  - It may involve other statistical tests
- Interpret the results

# Quantitative variables

- We are interested to assess the performance of an automatic blood pressure monitor, so we'll compare the results given by this device with the results given by a manual blood pressure monitor.

- The automatic blood pressure monitor is NOT CONFORM if its results vary more than 10% of the results given by a manual blood pressure monitor.

- In order to do that we'll measure the blood pressure in a sample of 98 persons with both devices - variable: SBP/DBP – continuous quantitative variable

- $H_0$ : there is no difference between the results given by the automatic blood pressure monitor and the results given by a manual blood pressure monitor.

- $H_1$ : there is difference (more than 10%) between the results given by the automatic blood pressure monitor and the results given by a manual blood pressure monitor.

- Which statistical test should we use?





# Quantitative variables

Data follow normal distribution

- **Z**
- **T**
- **ANOVA**
- ...

Parametrical tests

Data not follow normal distribution

- **Kruskal-Wallis**
- **Mann-Whitney U**
- **Mann-Whitney-Wilcoxon**
- **Wilcoxon signed-rank**
- ...

Non - Parametrical tests

# Quantitative variables

- We are interested to assess the performance of an automatic blood pressure monitor, so we'll compare the results of that with the results given by a manual blood pressure monitor.

- The automatic blood pressure monitor is NOT CONFORM if its results vary more than 10% of the results given by a manual blood pressure monitor.

- In order to do that we'll measure the blood pressure in a sample of 98 persons with both devices - variable: SBP/DBP – continuous quantitative variable
- Should we use a parametrical test, or should we use a non-parametrical test?





# Quantitative variables

A **normality test** is used to determine whether sample data has been drawn from a normally distributed population (within some tolerance).



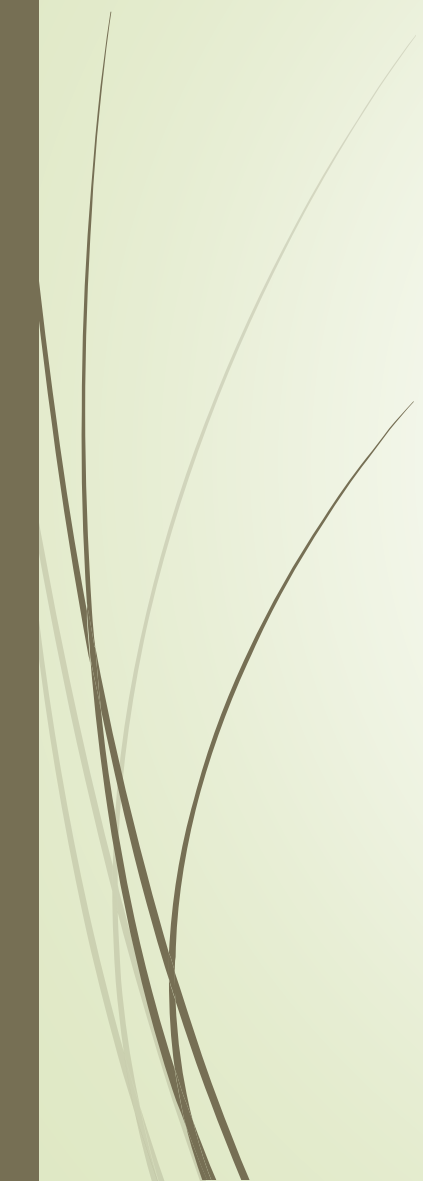


# Normality tests

- Kolmogorov-Smirnov (only if sample size  $\geq 50$ )
- Shapiro-Wilk
- Test the internal quality of the sample



# Normality tests - Hypotheses

- **H0: There is no difference between the sample distribution and normal distribution**
  - **H1: There is difference between the sample distribution and normal distribution**
- 



# Normality tests - decision

The result of the test:  $p$

If  $p \leq 0,05$   $H_0$  can be rejected,  $H_1$  could be accepted with 95% of confidence – the sample distribution is different than normal distribution

If  $p > 0,05$   $H_0$  cannot be rejected, we cannot demonstrate that the sample distribution is different than normal distribution

# Example - decision



- $H_0$ : There is no difference between the distribution of the data in the sample and the normal distribution
- $H_1$ : There is difference between the distribution of the data in the sample and the normal distribution
- We apply Kolmogorov-Smirnov (sample size=98  $\geq$  50)
- $p=0,03$
- Because  $p < 0,05$  we can reject  $H_0$  and accept  $H_1$  with 95% of confidence, so with 95% of confidence, there is difference between the distribution of the data in the sample and the normal distribution, so we must use a non-parametrical statistical test







- We decide to repeat the experiment, this time we have a sample size 32
- We apply Shapiro-Wilk test (sample size=32<50)
- $p=0,09$
- Because  $p>0,05$  we cannot reject  $H_0$ , we cannot demonstrate that there is difference between the distribution of the data in the sample and the normal distribution, so we must use a parametrical statistical test







# Conclusion

- When we have to analyze a sample according to a quantitative variable we must decide if we'll use parametrical or non-parametrical tests. For that we must check if the data distribution is similar with normal distribution using normality tests (Kolmogorov-Smirnov or Shapiro-Wilk test).
- If  $p > 0,05$ , parametrical tests should be used
- If  $p \leq 0,05$  non-parametrical tests should be used



# Remember!

- The result of the normality test is only used to decide what kind of test you'll use to test the statistical hypothesis!
- The normality test will be always followed by another statistical test – parametrical or non-parametrical



# Parametrical tests

- Quantitative normal distributed variables
- The estimator for the values of the variable of interest is the mean



# Z test , two samples, for means

- Conditions of application
  - Normal distributed data
  - The variance in population is known (or at least it can be estimated)
  - Big samples (sample size  $> 30$ )
- Null hypotheses:  $m_1 = m_2$
- Alternative hypotheses:  $m_1 \neq m_2$  (two tail test) or  $m_1 > m_2$  (one tail test)

# Problem

- A trainer is interested to check that a new training method leads to a better performance on long-jump challenge. For this, he compared the results of a group of 50 athletes trained with this new method - average of the jumps  $m_1 = 730$  cm- with the results of a control group (size 83) – average of the jumps  $m_2 = 710$  cm. The data are normal distributed. Usually, on long-jump, the expected variance is 30 cm. The confidence level is set to 95%.
- Conditions for Z test
  - Normal distributed data - YES
  - The variance in population is known (or at least it can be estimated) – YES – 30 cm
  - Big samples (sample size  $> 30$ ) YES – sample size =  $50 + 83 = 133$



# Applying z test

- $H_0$ : there is no significant difference between the performance on long-jump challenge in the two groups ( $m_1$  is not significant different than  $m_2$ )
- $H_1$ : the new method of training increase the performance on long-jump challenge ( $m_1 > m_2$ ) – one tail test
- Apply the Z test  $\Rightarrow p=0,005$
- $P < 0,05$  so with 95% of confidence we can say that the new method of training increase the performance on long-jump challenge



# T test (student test) – two samples

- Conditions of application
  - Normal distributed data
- Null hypotheses:  $m_1 = m_2$
- Alternative hypotheses:  $m_1 \neq m_2$  (two tail test) or  $m_1 > m_2$  (one tail test)



# Are the samples independent?

Yes - T test for independent samples

- In order to test the efficacy of a drug in reducing the value of triglycerides we compare the level of the triglycerides of a group of people which had used that drug with the level of the triglycerides of a group of people which had used placebo

No - T test for paired samples

- In order to test the efficacy of a drug in reducing the value of triglycerides we measure the value of triglycerides before and after the treatment



# T test for paired samples

- In order to test the efficacy of a drug in reducing the value of triglycerides we measure the value of triglycerides before and after the treatment (95% confidence)
- $H_0$ : There is no significant difference between the mean of the TG before and after the treatment
- $H_1$ : The mean of TG after the treatment is significantly lower than the mean of TG before treatment (one tail test)

Applying the test  $\Rightarrow p=0,0000000005$

- $P < 0,05$ , we reject  $H_0$ , so, with 95% of confidence, the drug is efficient in reducing TG

TG before mg/dL	TG after mg/dL
304	154
302	152
251	221
426	296
387	157
296	126
293	223
488	438
383	313
432	282
341	211
214	164
201	100
484	254
342	232
409	219
353	183
223	73
224	71

# T test for independent samples

In order to test the efficacy of a drug in reducing the value of triglycerides we compare the level of the triglycerides of a group of people which had used that drug with the level of the triglycerides of a group of people which had used placebo

There are two different T tests for independent samples (T test assuming equal variances and T test assuming unequal variances) and we must choose which test is appropriate

The decision will be given by the result of another statistical test (F test for variances / Barlett test / Levine Test)



# F test for variances / Barlett test / Levine Test

- $H_0$ : There is no significant difference between the variances of the two groups (the variances are equal)
- $H_1$ : There is significant difference between the variances of the two groups (the variances are unequal)
- If  $p \leq 0,05$  we will apply T test for unequal variances
- If  $p > 0,05$  we will apply T test for equal variances

Example - In order to test the efficacy of a drug in reducing the value of triglycerides we compare the level of the triglycerides of a group of people which had used that drug with the level of the triglycerides of a group of people which had used placebo – 95% confidence

- $H_0$ : There is no significant difference between the mean of the TG when we use drug or when we use placebo
- $H_1$ : The mean of TG when we use drug is significantly lower than the mean of TG when we use placebo (one tail test)
- First, we apply F test  $\Rightarrow p=0,39 > 0,05$ , so we must use T test for equal variances
- We apply T test for equal variances  $\Rightarrow p=0,36 > 0,05$  – we cannot demonstrate that this drug is more efficient than placebo in reducing the TG value

TG after drug mg/dL
154
152
221
296
157
126
223
438
313
282
211
164
100
183
73
71

TG after placebo mg/dL
169
152
226
301
152
141
238
453
308
287
216
164
95
254
242
224
198
88
66

	TG after drug mg/dL	TG after placebo mg/dL
Mean	197,75	209,1578947



# Remember

- In order to apply T-test for independent samples you must first test the equality of the variances (F test / Levine test / Barlet test)
- The test for the equality of the variances (TEV) will be always followed by a T-test for independent samples as follows:
  - T-test for independent samples assuming equal variances when the result of TEV  $p > 0,05$
  - T-test for independent samples assuming un-equal variances when the result of TEV  $p \leq 0,05$

# Be Prepared for the next lecture!

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